FPGA-Based Design and Implementation of a Multi-Gbps LDPC Decoder

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LDPC Codes

- LDPC codes: Forward Error Correction (FEC) codes that exhibit excellent error correction performance.
- Adopted by many present and future standards.
- Hardware-friendly due to inherent parallelism of decoding algorithms.
- FPGAs can support multiple standards and rates via runtime reconfiguration.

Channel Model LDPC Codes Decoding Algorithm

Channel Model

Memoryless AWGN channel:

$$y_i = x_i + n_i, \quad n_i \sim \mathcal{N}(0, \sigma^2).$$

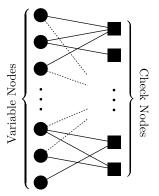
Decoding consists of finding most likely x_i, based on y, for all i = 1,..., n:

$$\hat{x}_i = \arg \max_{x_i} p(x_i | \mathbf{y}) \longleftarrow \mathsf{NP-hard!}$$

Channel Model LDPC Codes Decoding Algorithm

LDPC Codes

- Defined through a parity-check matrix H.
- Represented by a Tanner graph \longrightarrow
- Decoding via Min-Sum message passing on the graph.



Channel Model LDPC Codes Decoding Algorithm

Min-Sum Decoding Algorithm

• Variable-to-check:
$$L_{ij} = \overbrace{2y_i/\sigma^2}^{\text{Initial LLR}} + \sum_{k \in \mathcal{C}(i)/j} R_{ki}.$$

• Check-to-variable: $R_{ij} = \left(\prod_{k \in \mathcal{V}(i)/j} \operatorname{sign}(L_{ki})\right) \min_{k \in \mathcal{V}(i)/j} |L_{ki}|.$

• A maximum of *k* iterations is performed.

Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

LDPC Decoder Architectures

Fully Parallel: Every VN and CN represented in hardware.

- Very high throughput.
- High hardware utilization, very complex routing.
- Serial: One VN and one CN.
 - Low throughput.
 - Very efficient hardware utilization, minimal routing.
- **Partially Parallel**: **Compromise** between the two.

Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

Message Quantization

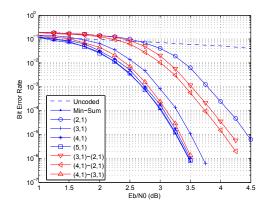
- (*n*, *m*) signed fixed point quantization:
 - Total of *n* bits for each message.
 - *m* bits are used for fractional part.

• $(n_1, m_1)-(n_2, m_2)$ hybrid quantization:

- (n_1, m_1) quantization for initial LLR messages.
- (n₂, m₂) quantization for variable-to-check and check-to-variable messages.
- By choosing *n*₂ < *n*₁, routing and processing unit complexity can be **significantly reduced**.

Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

Effect of Message Quantization on Performance



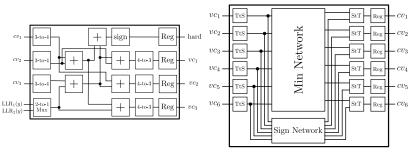
Comparison of (4,1) with hybrid:

- (4,1)-(3,1): negligible loss, -25% wires, -45% LUTs.
- (3,1)-(2,1): 0.75 dB loss, -50% wires, -71% LUTs.

Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

Variable and Check Node Processing Units

Straightforward combinational logic implementing the variable and check node update rules.



Variable node.

Check node.

Message Quantization Variable and Check Node Processing Units **Pipelining** Early Termination

Pipelining

- Usually, one decoding iteration is considered to last one clock cycle \rightarrow high path delays.
- **Idea**: add registers to reduce path delays.
- **Problem**: decoding now takes twice as many cycles.
- **Observation**: at each cycle, either VNs or CNs are idle.
- **Solution**: decode two codewords simultaneously.

Input	Input	Bubble	VN	CN	VN	CN	Output	Output						
		Input	Input	VN	CN	VN	CN	Bubble	Output	Output				
				Input	Input	Bubble	VN	CN	VN	CN	Output	Output		
-	$\xrightarrow{\iota}$					Input	Input	VN	\mathbf{CN}	VN	CN	Bubble	Output	Output

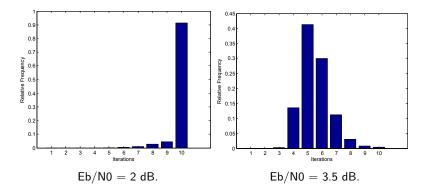
Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

Early Termination

- If after some iteration we have reached a valid codeword, decoding can halt.
- At high Eb/N0, this can lead to significant increase in average throughput.
- Significant I/O problems due to non-uniform distribution of required iterations.

Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

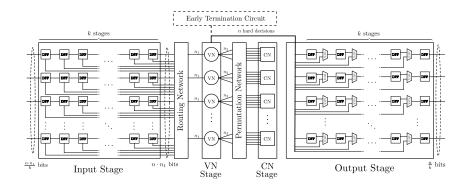
Early Termination



- **Idea**: force decoder to perform at least k/2 iterations.
 - Small impact on throughput.
 - *k* guaranteed cycles for I/O.

Message Quantization Variable and Check Node Processing Units Pipelining Early Termination

Overall Datapath



Results

	This work (4,1)–(3,1)	Chandrasetty & Aziz (2011)	gain/ loss
Decoding Algorithm	Min-Sum	MMS	
Clock Frequency	154.30 MHz	149.00 MHz	3.5%
$Eb/N0$ at 10^{-6} BER	3.5 dB	4 dB	0.50 dB
Av. Iter. at 10^{-6} BER	5.8	6.8	14.7%
Av. Throughput	14.6 Gbps	12.6 Gbps	15.9%
LUT Utilization	89.4%	98.5%	9.1%
Max. Delay	220 ns	121 ns	81.8%

Results

	This work (3,1)–(2,1)	Chandrasetty & Aziz (2011)	gain/ loss
Decoding Algorithm	Min-Sum	MMS	
Clock Frequency	211.40 MHz	149.00 MHz	41.9%
$Eb/N0$ at 10^{-6} BER	4.25 dB	4 dB	0.25 dB
Av. Iter. at 10^{-6} BER	5.6	6.8	17.6%
Av. Throughput	21.6 Gbps	12.6 Gbps	71.4%
LUT Utilization	47.6%	98.5%	50.9%
Max. Delay	161 ns	121 ns	33.1%

Conclusion

We presented an FPGA-based LDPC decoder architecture which:

Outperforms the state of the art by:
15.9% at a 0.50 dB lower Eb/N0.
71.4% at a 0.25 dB higher Eb/N0.

2 Requires 9.1% and 50.9% less logic, respectively.

3 Fully addresses the I/O problems due to early termination.



Questions?